Chapter I

1)

We start by calculating the resistance, adjusting the values for 90°C and considering skin and proximity effects for 50Hz.

$$\rho = \rho_{Cu}\frac{\pi R_1^2}{S} \Leftrightarrow \rho = 1.724\cdot 10^{-8}\frac{\pi\cdot 16.85^2}{800} \Leftrightarrow \rho = 1.922\cdot 10^{-8}\ \left[\Omega.m\right] \quad (1.1)$$

$$R_{DC} = \frac{\rho}{S} \Leftrightarrow R_{DC} = \frac{1.922\cdot 10^{-8}}{\pi\left(16.85\cdot 10^{-3}\right)^2} \Leftrightarrow R_{DC} = 21.55\cdot 10^{-6}\ \left[\Omega.\text{m}^{-1}\right] \quad (1.2)$$

$$R_{DC}\left(90°\right) = 21.55\cdot 10^{-6}\left(1+3.93\cdot 10^{-3}\left(90-20\right)\right) \Leftrightarrow R_{DC}\left(90°\right) = 27.48\cdot 10^{-6}\ \left[\Omega.m^{-1}\right] \quad (1.3)$$

$$x_s^2 = \frac{8\pi f}{R_{DC}}10^{-7}k_s \Leftrightarrow x_s^2 = \frac{8\pi 50}{27.48\cdot 10^{-6}}10^{-7}\cdot 1 \Leftrightarrow x_s^2 = 4.573 \quad (1.4)$$

$$y_s = \frac{x_s^4}{192+0.8x_s^4} \Leftrightarrow y_s = 0.1 \quad (1.5)$$

$$y_p = y_s\left(\frac{d_c}{s}\right)^2\cdot\left[0.312\left(\frac{d_c}{s}\right)^2+\frac{1.18}{y_s+0.27}\right] \Leftrightarrow$$
$$\Leftrightarrow y_p = 0.1\cdot\left(\frac{33.7}{1000}\right)^2\cdot\left[0.312\left(\frac{33.7}{1000}\right)^2+\frac{1.18}{0.1+0.27}\right] \quad (1.6)$$
$$\Leftrightarrow y_p = 0.00036$$

Notice that the proximity effect is much lower than for an equivalent cable installed in a trefoil formation, because of the largest distance between the cables.

$$R_{50Hz} = 27.8\cdot 10^{-6}\left(1+0.1+0.00036\right) \Leftrightarrow R_{50Hz} = 30.59\cdot 10^{-6}\ \left[\Omega.\text{m}^{-1}\right] \quad (1.7)$$

The reactance of the conductor can be easily calculated:

$$X_L = 2\pi 50\frac{\mu}{2\pi}\ln\left(\frac{932}{16.85\cdot 10^{-3}e^{-\frac{1}{4}}}\right) \Leftrightarrow X_L = 7.02\cdot 10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \quad (1.8)$$

The next step is to calculate the parameters of the screen:

$$R_S = \frac{\rho}{S} \Leftrightarrow R_S = \frac{2.826\cdot 10^{-8}}{600\cdot 10^{-6}} \Leftrightarrow R_S = 47.1\cdot 10^{-6}\ \left[\Omega.\text{m}^{-1}\right] \quad (1.9)$$

The temperature of the screen is lower than 90°C, but in order to simplify the process we consider 90°C as being the temperature in the screen.

$$R_{S,90°} = R_S\left(1+\alpha_T\left(T-20\right)\right) \Leftrightarrow R_{S,90°} = 47.1\cdot 10^{-6}\left(1+4.03\cdot 10^{-3}\left(90-20\right)\right) \Leftrightarrow R_{S,90°} = 60.4\cdot 10^{-6}\ \left[\Omega.m^{-1}\right] \quad (1.10)$$

$$X_S = \frac{\omega\mu}{2\pi}\ln\left(\frac{D_e}{\frac{R_2+R_3}{2}}\right) \Leftrightarrow X_S = 50\mu\ln\left(\frac{932}{\frac{40.76\cdot 10^{-3}+38.35\cdot 10^{-3}}{2}}\right) \Leftrightarrow X_S = 6.33\cdot 10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \quad (1.11)$$

We are now ready to calculate the series impedance of the conductor and screen:

$$Z_{Self} = R_{50Hz} + R_e + jX_L \Leftrightarrow Z_{Self} = 30.59\cdot10^{-6} + 49.35\cdot10^{-6} + j7.4\cdot10^{-4} \Leftrightarrow$$
$$\Leftrightarrow Z_{Self} = 79.94\cdot10^{-6} + j7.4\cdot10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \tag{1.12}$$

$$Z_{Self,S} = R_S + R_e + jX_S \Leftrightarrow Z_{Self,S} = 47.1\cdot10^{-6} + 49.35\cdot10^{-6} + j6.33\cdot10^{-4}$$
$$\Leftrightarrow Z_{Self,S} = 96.45\cdot10^{-6} + j6.33\cdot10^{-4}\left[\Omega.\text{m}^{-1}\right] \tag{1.13}$$

The mutual impedance between phases is calculated in three steps. It is first calculated the mutual impedance between adjacent phases, then between the outer phases and finally it is done the average:

$$Z_{M\_i} = R_e + j\frac{\omega\mu}{2\pi}\ln\left(\frac{D_e}{s}\right) \Leftrightarrow Z_{M\_i} = 49.35\cdot10^{-6} + j50\mu\ln\left(\frac{932}{1}\right) \Leftrightarrow Z_{M\_i} = 49.35\cdot10^{-6} + j4.30\cdot10^{-4}\left[\Omega.\text{m}^{-1}\right] \tag{1.14}$$

$$Z_{M\_o} = R_e + j\frac{\omega\mu}{2\pi}\ln\left(\frac{D_e}{2s}\right) \Leftrightarrow Z_{M\_o} = 49.35\cdot10^{-6} + j50\mu\ln\left(\frac{932}{2}\right) \Leftrightarrow Z_{M\_i} = 49.35\cdot10^{-6} + j3.86\cdot10^{-4}\left[\Omega.\text{m}^{-1}\right] \tag{1.15}$$

$$Z_M = \frac{2Z_{M\_i} + Z_{M\_o}}{3} \Leftrightarrow Z_M = \frac{2\left(49.35\cdot10^{-6} + j4.30\cdot10^{-4}\right) + 49.35\cdot10^{-6} + j3.86\cdot10^{-4}}{3}$$
$$\Leftrightarrow Z_M = 49.35\cdot10^{-6} + j4.15\cdot10^{-4}\left[\Omega.\text{m}^{-1}\right] \tag{1.16}$$

Mutual inductance between conductor and screen:

$$Z_{M,S} = R_e + jX_S \Leftrightarrow Z_{M,S} = 49.35\cdot10^{-6} + j6.33\cdot10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \tag{1.17}$$

The Positive-sequence and zero-sequence impedances can now be calculated:

$$Z^+ = \left(Z_{Self} - Z_M\right) - \frac{\left(Z_{M,S} - Z_M\right)^2}{Z_{Self,S} - Z_M} \Leftrightarrow Z^+ = 75.59\cdot10^{-6} + \text{j}1.17\cdot10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \tag{1.18}$$

$$Z^0 = Z_{Self} + 2Z_M - \frac{\left(Z_{M,S} + 2Z_M\right)^2}{Z_{Self,S} + 2Z_M} \Leftrightarrow Z^0 = 77.49\cdot10^{-6} + j1.08\cdot10^{-4}\ \left[\Omega.\text{m}^{-1}\right] \tag{1.19}$$

2)

We start by calculating the thermal resistances for the cable.

$$T_1 = \frac{\rho_T}{2\pi}\ln\left(1+\frac{2(R_2-R_1)}{2R_1}\right) \Leftrightarrow T_1 = \frac{3.5}{2\pi}\ln\left(1+\frac{2(38.35-16.85)}{2\cdot 16.85}\right) \Leftrightarrow T_1 = 0.458 \text{ m}^2K\cdot W^{-1} \quad (1.20)$$

$$T_2 = \frac{\rho_T}{2\pi}\ln\left(1+\frac{2(R_4-R_3)}{2R_4}\right) \Leftrightarrow T_2 = \frac{3.5}{2\pi}\ln\left(1+\frac{2(44.75-40.76)}{2\cdot 44.75}\right) \Leftrightarrow T_2 = 0.0476 \text{ m}^2K\cdot W^{-1} \quad (1.21)$$

$$T_3 = 0 \text{ m}^2K\cdot W^{-1} \quad (1.22)$$

The calculation of the external thermal resistance is divided into several steps:

$$u = \frac{2h}{2R_4} \Leftrightarrow u = \frac{2\cdot 1}{2\cdot 44.75\cdot 10^{-3}} \Leftrightarrow u = 22.35 \quad (1.23)$$

The hottest cable in this configuration is the one installed in the middle (we assume that there are not any other external heat sources). Thus, we do the calculation for that cable:

$$F_B = \left(\frac{d_{21}^{'}}{d_{21}}\right)\cdot\left(\frac{d_{23}^{'}}{d_{23}}\right) \Leftrightarrow F_B = \frac{\sqrt{s^2+(2h)^2}}{s}\cdot\frac{\sqrt{s^2+(2h)^2}}{s} \Leftrightarrow F_B = \frac{\sqrt{1^2+(2\cdot 1)^2}}{1}\cdot\frac{\sqrt{1^2+(2\cdot 1)^2}}{1} \Leftrightarrow F_B = 5 \quad (1.24)$$

We can finally calculate the external thermal resistance:

$$T_4 = \frac{\rho_T}{2\pi}\ln\left(\left(\text{u}+\sqrt{\text{u}^2-1}\right)F_B\right) \Leftrightarrow T_4 = \frac{0.9}{2\pi}\ln\left(\left(22.35+\sqrt{22.35^2-1}\right)\cdot 5\right) \Leftrightarrow T_4 = 0.775 \text{ m}^2K\cdot W^{-1} \quad (1.25)$$

Before calculating the ampacity we have to calculate the dielectric losses, which requires the calculation of the capacitance, and the ratio of losses in the screen of the cable.

$$\varepsilon = \varepsilon_{Ins}\frac{\ln\left(\frac{R_2}{R_1}\right)}{\ln\left(\frac{b}{a}\right)} \Leftrightarrow \varepsilon = 2.5\frac{\ln\left(\frac{38.35}{16.85}\right)}{\ln\left(\frac{37.35}{18.35}\right)} \Leftrightarrow \varepsilon^{'} = 2.89 \quad (1.26)$$

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{R_2}{R_1}\right)} \Leftrightarrow C = \frac{2\pi 2.89\cdot 8.85\cdot 10^{-12}}{\ln\left(\frac{38.35}{16.85}\right)} \Leftrightarrow C = 1.95\cdot 10^{-10} \ \left[\text{F.m}^{-1}\right] \quad (1.27)$$

$$W_d = \omega\cdot C\cdot U_0^2\cdot\tan(\delta) \Leftrightarrow W_d = 2\pi 50\cdot 1.95\cdot 10^{-10}\cdot\left(165\cdot 10^3\right)^2 0.001 \Leftrightarrow W_d = 1.67 \text{ W.m}^{-1} \quad (1.28)$$

$$\lambda_1 = \frac{R_{S,90°}}{R_{50Hz}}\frac{1}{1+\left(\frac{R_S}{X_S}\right)^2} \Leftrightarrow \lambda_1 = \frac{60.4\cdot 10^{-6}}{30.59\cdot 10^{-6}}\frac{1}{1+\left(\frac{47.1\cdot 10^{-6}}{6.33\cdot 10^{-4}}\right)^2} \Leftrightarrow \lambda_1 = 1.96 \quad (1.29)$$

We are now in the possession of all required information and ready to calculate the ampacity:

$$I = \left[\frac{\Delta\theta - W_d\cdot\left[0.5\cdot T_1 + n\cdot(T_2+T_3+T_4)\right]}{R\cdot T_1 + n\cdot R\cdot(1+\lambda_1)\cdot T_2 + n\cdot R\cdot(1+\lambda_1+\lambda_2)\cdot(T_3+T_4)}\right]^{1/2} \quad (1.30)$$

$$I = \left[\frac{65-1.67\cdot\left[0.5\cdot 0.458+(0.0476+0.775)\right]}{30.59\cdot 10^{-6}\cdot 0.458+30.59\cdot 10^{-6}\cdot(1+1.96)\cdot 0.0476+30.59\cdot 10^{-6}\cdot(1+1.96)\cdot 0.775}\right]^{1/2} \Leftrightarrow I = 845.4A \quad (1.31)$$

3)

The only difference from this exercise to the previous exercise is in the calculation of the external thermal resistance T_4 (the mutual coupling is also affected, but it is not important for us).

$$u = \frac{2h}{2R_4} \Leftrightarrow u = \frac{2 \cdot \left(\frac{1+1.0775}{2} \right)}{2 \cdot 44.75 \cdot 10^{-3}} \Leftrightarrow u = 23.21 \tag{1.32}$$

$$T_4 = \frac{1.5\rho_T}{\pi}\left(\ln\left(2\mathrm{u}\right) - 0.630\right) \Leftrightarrow T_4 = \frac{1.5 \cdot 0.9}{\pi}\left(\ln\left(2 \cdot 23.21\right) - 0.630\right) \Leftrightarrow T_4 = 1.378 \ \mathrm{m}^2 K \cdot W^{-1} \tag{1.33}$$

$$I = \left[\frac{65 - 1.67 \cdot \left[0.5 \cdot 0.458 + (0.0476 + 1.378)\right]}{30.59 \cdot 10^{-6} \cdot 0.458 + 30.59 \cdot 10^{-6} \cdot (1+1.96) \cdot 0.0476 + 30.59 \cdot 10^{-6} \cdot (1+1.96) \cdot 1.378} \right]^{1/2} \Leftrightarrow I = 659.5A \tag{1.34}$$

We can see that to change from a flat to a trefoil formation represents a reduction of almost 200A in the cable ampacity.

4)

$$V_1 - V_2 = I\left(R + j\omega L\right)l \Leftrightarrow V_1 - V_2 = \left(V_2 j\frac{\omega C}{2}\right)l\left(R + j\omega L\right)l$$

$$\Leftrightarrow V_1 - V_2 = -V_2\frac{\omega^2 LCl^2}{2} + V_2 j\frac{\omega RCl^2}{2} \Leftrightarrow V_1 = V_2\left(1 - \frac{\omega^2 LCl^2}{2} + j\frac{\omega RCl^2}{2}\right) \tag{1.35}$$

$$\Leftrightarrow V_2 = V_1\frac{1}{1 - \frac{\omega^2 LCl^2}{2} + j\frac{\omega RCl^2}{2}}$$

The equation shows that there will be both a phase difference and a change in the magnitudes from V_1 to V_2.

$$\|V_2\| = \|V_1\|\frac{1}{\sqrt{\left(1 - \frac{\omega^2 LCl^2}{2}\right)^2 + \left(\frac{\omega RCl^2}{2}\right)^2}} \Leftrightarrow \|V_2\| = \|V_1\|\frac{1}{\sqrt{1 + \left(\frac{\omega^2 LCl^2}{2}\right)^2 - \omega^2 LCl^2 + \left(\frac{\omega RCl^2}{2}\right)^2}} \tag{1.36}$$

The denominator of the equation increases when the resistance increases. Thus, the voltage increase because of Ferranti effect is lower. If the resistance is large enough the voltage can even decrease from V_1 to V_2 (as often happens for OHLs).

Below is shown an example for both a typical cable and a typical OHL. Both are unloaded and as a result, in both cases the voltage increases from the sending end to the receiving end. However, it is possible to observe that the voltage increasing is much larger in the cable than in the OHL.

Parameters: Voltage at the sending end; 1pu
R_{Cable}=0.03475 Ω/km
L_{Cable}=0.0022 H/km
C_{Cable}=0.2242 µF/km

R_{OHL}=0.119 Ω/km
L_{OHL}=0.001276 H/km
C_{OHL}=0.0091 µF/km

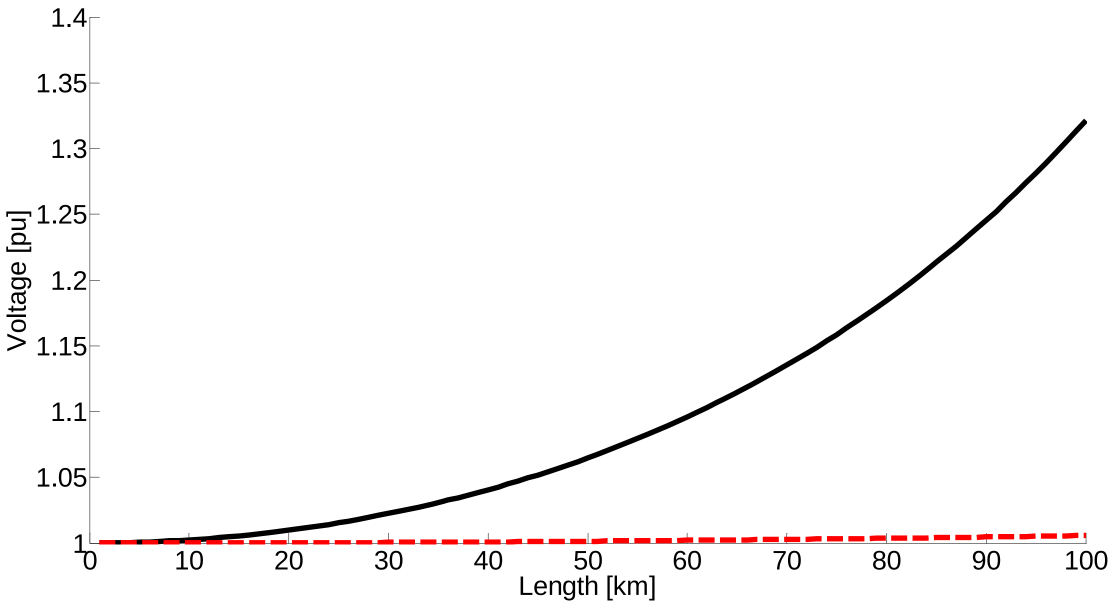

Fig. 1 Voltage in the receiving end of a line in function of the length. Solid line: Cable; Dashed line: OHL